Section 8.2 Solutions

1) $6x + 2y = 10$
$2x - y = 5$

Initial matrix

$$\left[\begin{array}{cc|c} 6 & 2 & 10 \\ \textcircled{2} & -1 & 5 \end{array}\right]$$

STEP 1: Create this ZERO

R1	6	2	10
$-3R2$	-6	3	-15
NEW R2	0	5	-5

Divide by 5, BEFORE creating new matrix

NEW R2	0	1	-1

2nd matrix

$$\left[\begin{array}{cc|c} 6 & \textcircled{2} & 10 \\ 0 & 1 & -1 \end{array}\right]$$

STEP 2: CREATE THIS 0

R1	6	2	10
$-2R2$	0	-2	2
NEW R1	6	0	12

DIVIDE by 6

NEW R1	1	0	2

Final matrix

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]$$

ANSWER
(2, -1)

3) $4x - 3y = -2$
$x - 5y = -9$

Initial matrix

$$\left[\begin{array}{cc|c} 4 & -3 & -2 \\ 1 & -5 & -9 \end{array}\right]$$

STEP 1: CREATE THIS 0

R1	4	-3	-2
-4R2	-4	20	36
NEW R2	0	17	34

DIVIDE by 17

NEW R2	0	1	2

SECOND MATRIX

$$\left[\begin{array}{cc|c} 4 & -3 & -2 \\ 0 & 1 & 2 \end{array}\right]$$

STEP 2: create this 0

R1	4	-3	-2
3R2	0	3	6
	4	0	4

DIVIDE by 4

NEW R1	1	0	1

FINAL MATRIX

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

ANSWER
(1, 2)

5) $5x + y = -7$
$3x - 2y = -12$

INITIAL MATRIX

$$\left[\begin{array}{cc|c} 5 & 1 & -7 \\ 3 & -2 & -12 \end{array}\right]$$

STEP 1: Create this 0

$-3R1$	-15	-3	21
$5R2$	15	-10	-60
NEW R2	0	-13	-39

Divide by -13

NEW R2 $\quad 0 \quad 1 \mid 3$

SECOND MATRIX

$$\left[\begin{array}{cc|c} 5 & 1 & -7 \\ 0 & 1 & 3 \end{array}\right]$$

STEP 2: CREATE THIS 0

$R1$	5	1	-7
$-1R2$	0	-1	-3
NEW R1	5	0	-10

Divide by 5

NEW R1 $\quad 1 \quad 0 \mid -2$

FINAL MATRIX

$$\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right]$$

ANSWER
$(-2, 3)$

7) $3x+2y=11$
$2x-y=5$

Initial matrix

$$\left[\begin{array}{cc|c} 3 & 2 & 11 \\ 2 & -1 & 5 \end{array}\right]$$ STEP 1: Create this 0

2R1	6	4	22
-3R2	-6	3	-15
NewR2	0	7	7

Divide by 7

NewR2	0	1	1

Second matrix

$$\left[\begin{array}{cc|c} 3 & 2 & 11 \\ 0 & 1 & 1 \end{array}\right]$$ STEP 2: Create this 0

R1	3	2	11
-2R2	0	-2	-2
NewR1	3	0	9

Divide by 3

NewR1	1	0	3

Final matrix

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$$

Answer
(3, 1)

9) $4x - 2y = 7$
$2x - 5y = -3$

Initial matrix

$$\left[\begin{array}{cc|c} 4 & -2 & 7 \\ \textcircled{2} & -5 & -3 \end{array}\right]$$ STEP 1: create this 0

R1	4	-2	7
-2R2	-4	10	6
NEW R2	0	8	13

Second matrix

$$\left[\begin{array}{cc|c} 4 & \textcircled{-2} & 7 \\ 0 & 8 & 13 \end{array}\right]$$ STEP 2: create this 0

4R1	16	-8	28
R2	0	8	13
NEW R1	16	0	41

Third matrix

$$\left[\begin{array}{cc|c} \frac{16}{16} & \frac{0}{16} & \frac{41}{16} \\ 0 & 8 & 13 \\ \frac{0}{8} & \frac{8}{8} & \frac{13}{8} \end{array}\right]$$ Divide R1 by 16
Divide R2 by 8

Final matrix $\left[\begin{array}{cc|c} 1 & 0 & 41/16 \\ 0 & 1 & 13/8 \end{array}\right]$

Answer $\left(\frac{41}{16}, \frac{13}{8}\right)$

11) $5x + 2y = 22$
$3x - 5y = 7$

INITIAL MATRIX

$$\left[\begin{array}{cc|c} 5 & 2 & 22 \\ 3 & -5 & 7 \end{array}\right]$$ STEP 1: CREATE THIS 0

-3R1	-15	-6	-66
5R2	15	-25	35
NEW R2	0	-31	-31

Divide by -31

NEW R2	0	1	1

SECOND MATRIX

$$\left[\begin{array}{cc|c} 5 & 2 & 22 \\ 0 & 1 & 1 \end{array}\right]$$ STEP 2: CREATE THIS 0

R1	5	2	22
-2R2	0	-2	-2
NEW R1	5	0	20

Divide by 4

1	0	4

FINAL MATRIX $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \end{array}\right]$

ANSWER (4, 1)

13)
$-x + y + 2z = 1$
$2x + 3y + z = -2$
$5x + 4y + 2z = 4$

INITIAL MATRIX

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ \textcircled{2} & 3 & 1 & -2 \\ \textcircled{5} & 4 & 2 & 4 \end{array}\right]$$

STEP 1: USE R1 & CREATE THESE 0'S

$R_1 \cdot 2$ $\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \end{array}\right]$ R_2

$R_1 \cdot 5$ $\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 5 & 4 & 2 & 4 \end{array}\right]$ R_2

$$\begin{array}{cccc} -2 & 2 & 4 & 2 \\ 2 & 3 & 1 & -2 \\ \hline 0 & 5 & 5 & 0 \end{array}$$

NEW R2

$$\begin{array}{cccc} -5 & 5 & 10 & 5 \\ 5 & 4 & 2 & 4 \\ \hline 0 & 9 & 12 & 9 \end{array}$$

DIVIDE BY 5

R2 $\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \end{array}\right]$

DIVIDE BY 3

$\left[\begin{array}{ccc|c} 0 & 3 & 4 & 3 \end{array}\right]$ NEW R3

2ND MATRIX

$$\left[\begin{array}{ccc|c} -1 & \textcircled{1} & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & \textcircled{3} & 4 & 3 \end{array}\right]$$

STEP 2: USE R2 & CREATE THESE 0'S

$R_1 - 1$ $\left[\begin{array}{ccc|c} -1 & \textcircled{1} & 2 & 1 \end{array}\right]$ R_2 $\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \end{array}\right]$

R_3 $\left[\begin{array}{ccc|c} 0 & \textcircled{3} & 4 & 3 \end{array}\right]$

$R_2 \cdot 3$ $\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \end{array}\right]$

$$\begin{array}{cccc} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 0 & -1 & -1 \end{array}$$

NEW R1

$$\begin{array}{cccc} 0 & 3 & 4 & 3 \\ 0 & -3 & -3 & 0 \\ \hline R_3\ 0 & 0 & 1 & 3 \end{array}$$

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(13) CONTINUED

3RD MATRIX

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

STEP 3: USE R3 TO CREATE THESE 0'S

R_3 0 0 1 | 3
R_1 1 0 -1 | -1

1 0 0 | 2

NEW R1

$R_3 - 1($ 0 0 1 | 3
R_2 0 1 1 | 0

R_3 0 0 -1 | -3
R_2 0 1 1 | 0

0 1 0 -3

NEW R2

FINAL MATRIX

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

(13) ANSWER $(2, -3, 3)$

15)

$-5x - y + 3z = -14$
$-2x + 2y - 6z = 16$
$x + 7y + 2z = -5$

Initial MATRIX

$$\left[\begin{array}{ccc|c} -5 & -1 & 3 & -14 \\ -2 & 2 & -6 & 16 \\ 1 & 7 & 2 & -5 \end{array}\right]$$

STEP 1
Use R1 to create these 0's

$R_1 \to$
$R_2 5$

$$\left[\begin{array}{ccc|c} -5 & -1 & 3 & -14 \\ -2 & 2 & -6 & 16 \end{array}\right]$$

R_1
$R_3 5$

$$\left[\begin{array}{ccc|c} -5 & -1 & 3 & -14 \\ 1 & 7 & 2 & -5 \end{array}\right]$$

$$\begin{array}{ccc|c} 10 & 2 & -6 & 28 \\ -10 & 10 & -30 & 80 \\ \hline 0 & 12 & -36 & 108 \end{array}$$

Divide by 12

$$\begin{array}{ccc|c} 0 & 1 & -3 & 9 \end{array}$$

NEW R2

R_1
R_3

$$\begin{array}{ccc|c} -5 & -1 & 3 & -14 \\ 5 & 35 & 10 & -25 \\ \hline 0 & 34 & 13 & -39 \end{array}$$

NEW Row 3

Secono MATRIX

$$\left[\begin{array}{ccc|c} -5 & -1 & 3 & -14 \\ 0 & 1 & -3 & 9 \\ 0 & 34 & 13 & -39 \end{array}\right]$$

STEP 2
Create 0's
Use R2

R_2
R_1

$$\begin{array}{ccc|c} 0 & 1 & -3 & 9 \\ -5 & -1 & 3 & -14 \\ \hline -5 & 0 & 0 & -5 \end{array}$$

Divide By -5

$$\begin{array}{ccc|c} 1 & 0 & 0 & 1 \end{array}$$

NEW Row 1

$R_2 -34$
R_3

$$\begin{array}{ccc|c} 0 & 1 & -3 & 9 \\ 0 & 34 & 13 & -39 \end{array}$$

R_2
R_3

$$\begin{array}{ccc|c} 0 & -34 & 102 & -306 \\ 0 & 34 & 13 & -39 \\ \hline 0 & 0 & 115 & -345 \end{array}$$

Divide By 115

$$\begin{array}{ccc|c} 0 & 0 & 1 & -3 \end{array}$$

NEW Row 3

(15) continued

THIRD MATRIX

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & 9 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

STEP 3: Create this 0 using R3

R_2 0 1 −3 9
R_3 3(0 0 1 −3

R_2 0 1 −3 9
R_3 0 0 3 −9
0 1 0 0 NEW R2

FINAL MATRIX

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

ANSWER (1, 0, −3)

17)

$-x + 2y - z = -17$

$2x - y + z = 21$

$3x + 2y + z = 19$

INITIAL MATRIX

$$\left[\begin{array}{ccc|c} -1 & 2 & -1 & -17 \\ 2 & -1 & 1 & 21 \\ 3 & 2 & 1 & 19 \end{array}\right]$$

STEP 1: USE R1 TO CREATE 0

$R_1 : 2$

$$\begin{array}{ccc|c} -1 & 2 & -1 & -17 \\ 2 & -1 & 1 & 21 \end{array} \quad R_2:$$

$R_1 : 3$

$$\begin{array}{ccc|c} -1 & 2 & -1 & -17 \\ 3 & 2 & 1 & 19 \end{array} \quad R_3:$$

$R_1:$

$$\begin{array}{ccc|c} -2 & 4 & -2 & -34 \\ 2 & -1 & 1 & 21 \\ \hline 0 & 3 & -1 & -13 \end{array}$$

NEW R2

$R_1:$

$$\begin{array}{ccc|c} -3 & 6 & -3 & -51 \\ 3 & 2 & 1 & 19 \\ \hline 0 & 8 & -2 & -32 \end{array}$$

DIVIDE BY 2

$$\begin{array}{ccc|c} 0 & 4 & -1 & -16 \end{array}$$

NEW R3

SECOND MATRIX

$$\left[\begin{array}{ccc|c} -1 & 2 & -1 & -17 \\ 0 & 3 & -1 & -13 \\ 0 & 4 & -1 & -16 \end{array}\right]$$

STEP 2 USE R2 TO CREATE 0

$R_1:$

$$\begin{array}{ccc|c} -1 & 2 & -1 & -17 \\ 0 & 3 & -1 & -13 \end{array} \quad \begin{matrix} \\ 2 \end{matrix}$$

$R_1 : 3$

$R_2 : 2$

$R_2:$

$$\begin{array}{ccc|c} 0 & 3 & -1 & -13 \\ 0 & 4 & -1 & -16 \end{array} \quad \begin{matrix} -4 \\ 3 \end{matrix}$$

$R_1:$

$$\begin{array}{ccc|c} 3 & -6 & 3 & 51 \\ 0 & 6 & -2 & -26 \\ \hline 3 & 0 & 1 & 25 \end{array}$$

NEW ROW 1

$R_2:$

$$\begin{array}{ccc|c} 0 & -12 & 4 & 52 \\ 0 & 12 & -3 & -48 \\ \hline 0 & 0 & 1 & 4 \end{array}$$

NEW ROW 3

(7) continued

3RD MATRIX

$$\left[\begin{array}{ccc|c} 3 & 0 & 1 & 25 \\ 0 & 3 & -1 & -13 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

STEP 3: USE R3 TO CREATE 0's

$R_3 \cdot -1$:
R_1:

$$\begin{array}{ccc|c} 0 & 0 & 1 & 4 \\ 3 & 0 & 1 & 25 \end{array}$$

R_3:
R_2:

$$\begin{array}{ccc|c} 0 & 0 & 1 & 4 \\ 0 & 3 & -1 & -13 \end{array}$$

$$0 \quad 3 \quad 0 \quad -9$$

DIVIDE BY 3

$$0 \quad 1 \quad 0 \quad -3$$

NEW ROW 2

R_3:
R_1:

$$\begin{array}{ccc|c} 0 & 0 & -1 & -4 \\ 3 & 0 & 1 & 25 \end{array}$$

$$3 \quad 0 \quad 0 \quad 21$$

DIVIDE BY 3

$$1 \quad 0 \quad 0 \quad 7$$

NEW R1

FINAL MATRIX

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

ANSWER (7, -3, 4)

19)

$4x + y + z = 9$

$3x - 2y + z = 4$

$5x - 4y + z = 6$

Initial matrix

$$\left[\begin{array}{ccc|c} 4 & 1 & 1 & 9 \\ 3 & -2 & 1 & 4 \\ 5 & -4 & 1 & 6 \end{array}\right]$$

STEP 1
CREATE 0's
USE R1

R_1: 4 1 1 | 9 } 3
R_2: 3 -2 1 | 4 } -4

R_1: 4 1 1 | 9 } 5
R_3: 5 -4 1 | 6 } -4

R_1: 12 3 3 | 27
R_2: -12 8 -4 | -16
0 11 -1 | 11

NEW Row 2

R_1: 20 5 5 | 45
R_3: -20 16 -4 | -24
0 21 1 | 21

NEW Row 3

Second matrix

$$\left[\begin{array}{ccc|c} 4 & 1 & 1 & 9 \\ 0 & 11 & -1 & 11 \\ 0 & 21 & 1 & 21 \end{array}\right]$$

STEP 2
CREATE 0's
USE R2

R_2: 0 11 -1 | 11
R_1: 4 1 1 | 9

R_2: 0 11 -1 | 11 } -21
R_3: 0 21 1 | 21 } 11

R_2: 0 11 -1 | 11
R_1: -44 -11 -11 | -99
-44 0 -12 | -88

DIVIDE By -4

11 0 3 | 22

NEW Row 1

R_3: 0 -231 21 | -231
R_3: 0 231 11 | 231
0 0 32 | 0

DIVIDE By 32

0 0 1 | 0

NEW Row 3

(19) continued

THIRD MATRIX

$$\left[\begin{array}{ccc|c} 11 & 0 & 3 & 22 \\ 0 & 11 & -1 & 11 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

STEP 3: CREATE 0'S USE ROW 3

$R_3: -3$
$R_1:$

$$\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 11 & 0 & 3 & 22 \end{array}$$

$R_3:$
$R_2:$

$$\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 11 & -1 & 11 \\ \hline 0 & 11 & 0 & 11 \end{array}$$

DIVIDE by 11

$$0 \quad 1 \quad 0 \mid 1$$

NEW ROW 2

$R_3:$
$R_1:$

$$\begin{array}{ccc|c} 0 & 0 & -3 & 0 \\ 11 & 0 & 3 & 22 \\ \hline 11 & 0 & 0 & 22 \end{array}$$

DIVIDE by 11

$$1 \quad 0 \quad 0 \mid 2$$

NEW ROW 1

FINAL MATRIX

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

ANSWER $(2, 1, 0)$